St Benedict's HS Cluster

Numeracy

A Guide for pupils, parents and teachers to support good practice and consistent technique for the teaching of Numeracy Topics.

Updated 2017

That's not how I did that at school!

Things are laid out so differently!

Introduction

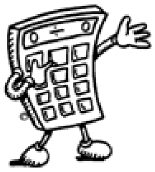
Welcome to our Numeracy booklet! We hope that within this booklet you will find support and understanding for the development of good Numeracy technique and practice. By using and referring to the information in this booklet, it will lead to a more consistent approach in the teaching of Numeracy topics across our Primary cluster and within our Secondary school.

Strength in Numeracy is the foundation for strength in Maths and in many other subject areas. We hope this booklet helps to develop that strength!

Table of Contents

Addition

Mental Strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example 1 Calculate 54 + 27

Partitioning Add tens, then add units, then add together

50 + 20 = 70 4 + 7 = 11 70 + 11 = 81

Chunking Split up number to be added into tens and units and add separately

54 + 20 = 74 74 + 7 = 81

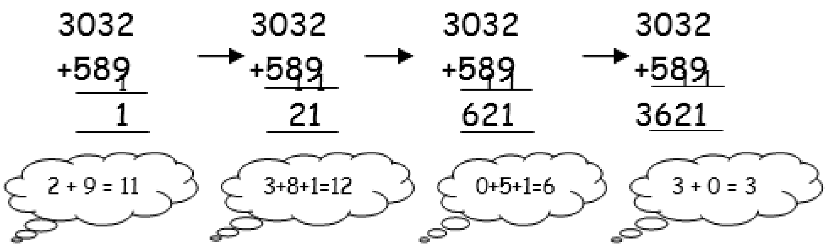
Compensating Round up to nearest 10, then subtract

54 + 30 = 84 but 30 is too much so subtract 3
84 – 3 = 81

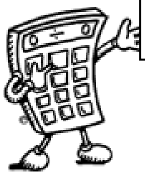
Formal Written Method

When adding numbers, ensure that the numbers are linked up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589



Subtraction



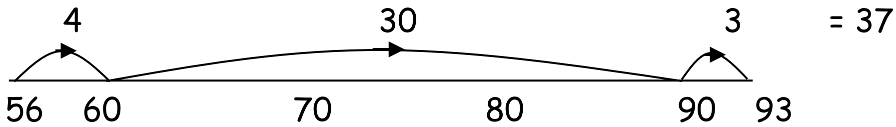
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate 93 - 56

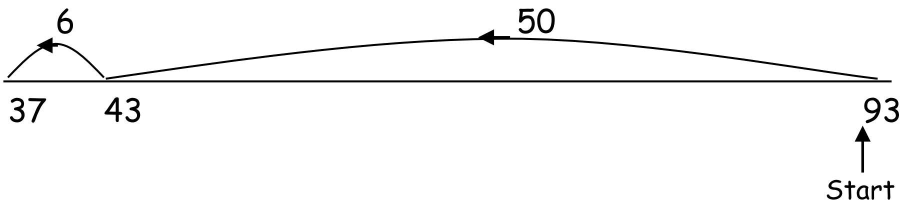
Adding Up/Bridging through 10

Count on from 56 until you reach 93. This can be done in several ways e.g.



Chunking Break up the number being subtracted

e.g. subtract 50, then subtract 6 93 - 50 = 43
43 - 6 = 37



Written Method

Example 1 4590 – 386

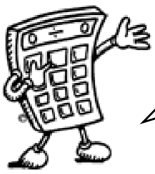
			8	1
	4	5	~~9~~	0
-		3	8	6
	4	2	0	4

We do not "borrow and pay back".

Example 2 Subtract 692 from 14597

	3	1			
	~~1~~	4	5	9	7
-			6	9	2
	1	3	9	0	5

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 10. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

Method 2

40 x 6
=240

40 is 1 too many
so take away 6x1

240 - 6
= 234

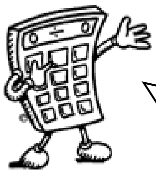
Written Method

(c) 436 x 6

```
  4 3 6
x  2 3 6
-------
2 6 1 6
```

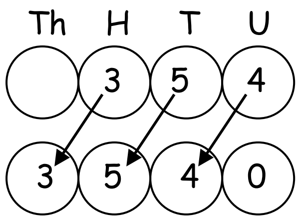
Multiplication 2

Multiplying by multiples of 10, 100 and 1000

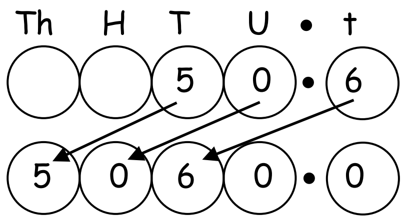


To multiply by **10** you move every digit ***one*** place to the left.
To multiply by **100** you move every digit ***two*** places to the left.
To multiply by **1000** you move every digit ***three*** places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



354 x 10 = 3540



50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10. *

35 x 3 = 105
105 x 10 = 1050

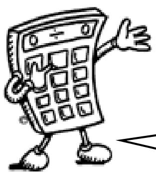
so 35 x 30 = 1050

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100.

436 x 6 = 2616
2616 x 100 = 261600

so 436 x 600 = 261600



We may also use these rules for multiplying decimal numbers.

Example 2 (a) 2·36 x 20

2·36 x 2 = 4.72
4·72 x 10 = 47.2

so 2·36 x 20 = 47·2

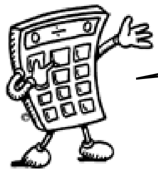
(b) 38·4 x 50

38·4 x 5 = 192.0
192·0x 10 = 1920

so 38·4 x 50 = 1920

*** It is also correct to multiply by 10 first and then by 3**

Division



You should be able to divide by a single digit or by a multiple of 10, 100 or 1000 without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r|l} & \;\;2\;4 \\ \hline 8 & \not{1}\,{}^{1}9\,{}^{3}2 \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r|l} & 1 \cdot 5\;8 \\ \hline 3 & 4 \cdot {}^{1}7\,{}^{2}4 \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r|l} & 0 \cdot 2\;7\;5 \\ \hline 8 & 2 \cdot {}^{2}2\,{}^{6}0\,{}^{4}0 \end{array}$$

Trailing zero

Each glass contains 0.275 litres

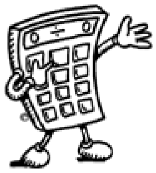
If you have a remainder at the end of a calculation, add "trailing zeros" onto the end of the decimal and continue with the calculation.

Order of Calculation (BODMAS)

Consider this: What is the answer to 2 + 5 x 8 ?

Is it 7 x 8 = 56 or 2 + 40 = 42 ?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS**

The **BODMAS** rule tells us which operations should be done first.
BODMAS represents:

(B)rackets
(O)perations
(D)ivide
(M)ultiply
(A)dd
(S)ubract

Scientific calculators use this rule; some basic calculators may not, so take care in their use.

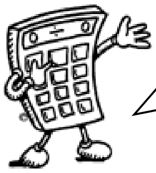
Example 1 $15 - 12 \div 6$ BODMAS tells us to divide first
$= 15 - 2$
$= 13$

Example 2 $9 + 5 \times 6$ BODMAS tells us to multiply first
$= 9 + 30$
$= 39$

Example 3 $(9 + 5) \times 6$ BODMAS tells us to work out the brackets first
$= 14 \times 6$
$= 84$

Example 4 $18 + 6 \div (5-2)$ Brackets first
$= 18 + 6 \div 3$ Then divide
$= 18 + 2$ Now add
$= 20$

Evaluating Formulae



To find the value of a variable in a formula, we must substitute all of the given values into the formula and then use BODMAS rules to work out the answer.

Example 1

Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

$P = 2L + 2B$	Step 1: write formula
$P = 2 \times 12 + 2 \times 7$	Step 2: substitute numbers and sign for letters
$P = 24 + 14$	Step 3: start to evaluate (BODMAS)
$P = 38$	Step 4: write answer

Example 2

Use the formula $I = \dfrac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$

$$I = \frac{V}{R}$$

$$I = \frac{240}{60}$$

$$I = 6$$

Example 3

Use the formula $F = 32 + 1{\cdot}8C$ to evaluate F when $C = 20$

$$F = 32 + 1{\cdot}8C$$

$$F = 32 + 1{\cdot}8 \times 20$$

$$F = 32 + 36$$

$$F = 68$$

Simple Equations – Cover Up

An equation is an expression which contains an equal sign.

For example: $x + 4 = 6$

$$x + 5 = 7$$
$$x = 2$$

$x = 2$ is the solution

$$y - 3 = 10$$
$$y = 13$$

<u>Cover up method</u>
Cover up the letter and look at the equation to see what number should be under your finger to make both sides equal.

$$2x + 1 = 17$$
$$\bigcirc + 1 = 17$$
$$2x = 16$$
$$2\bigcirc = 16$$
$$x = 8$$

Other examples with the Cover up method

$$3m - 4 = 11$$
$$\bigcirc - 4 = 11$$
$$3m = 15$$
$$3\bigcirc = 15$$
$$m = 5$$

$$\tfrac{1}{2}x + 3 = 6$$
$$\bigcirc + 3 = 6$$
$$\tfrac{1}{2}x = 3$$
$$\tfrac{1}{2}\bigcirc = 3$$
$$x = 6$$

As we have $\frac{1}{2}x = 3$, to find one full x we multiply the 3 by 2 (the denominator) to get 6

Harder Equations – Balancing

Another way of balancing/solving equations is when you keep the equation balanced by doing the same thing to both sides.

Example 1

$$x + 1 = 10$$
$$-1 \quad -1$$
$$x = 9$$

← You subtract 1 from both sides

Example 2

$$3m - 4 = 11$$
$$+4 \quad +4$$
$$\frac{3m}{3} = \frac{15}{3}$$
$$m = 5$$

Example 3

$$\tfrac{1}{2}y + 6 = 8$$
$$-6 \quad -6$$
$$\tfrac{1}{2}y = 2$$
$$\times 2 \quad \times 2$$
$$y = 4$$

What happens if we have letters on both sides?

Example 4

$$2x = x + 4$$
$$-x \quad -x$$
$$x = 4$$

We need to alter the equation so that x only appears on one side of the equation.
Remove x from both sides

Example 5

$$5x = 12 + x$$
$$-x \quad -x$$
$$\frac{4x}{4} = \frac{12}{4}$$
$$x = 3$$

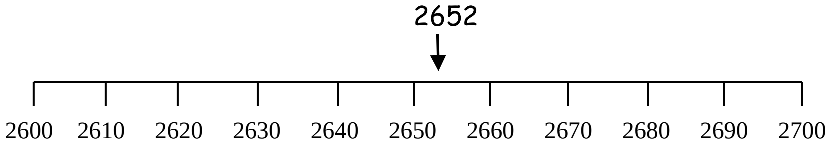
Example 6

$$7a + 16 = 9a$$
$$-7a \quad -7a$$
$$\frac{16}{2} = \frac{2a}{2}$$
$$8 = a$$
$$a = 8$$

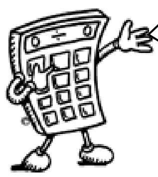
Letters do not always need to be kept on the LHS.

Estimation : Rounding

Numbers can be rounded to give an approximation.



26**5**2 rounded to the nearest 10 is 2650.
2**6**52 rounded to the nearest 100 is 2700. (2 figure accuracy)
2652 rounded to the nearest 1000 is 3000. (1 figure accuracy)



When rounding numbers which are exactly in the middle, convention is to **round up**.
78**6**5 rounded to the nearest 10 is 78**7**0.
If the number ends in **4 or below** -> **Round Down**
If the number ends in **5 or above** -> **Round Up**

The same principle applies to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example 1 Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

4**6** 753
= 47 000 to the nearest thousand

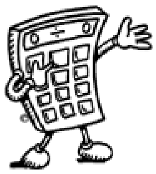
Example 2 Round 1·57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

1·5**7**359
= 1·57 to 2 decimal places

Estimation : Calculation

We can use rounded numbers to give us an estimated or approximate answer to a calculation. This allows us to check that our answer is sensible.



Example 1

Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

Estimate = 500 + 200 + 200 + 300 = 1200

Round each number to the highest place value.

Calculate:

```
 486
 205
 197
+321
1209
```

Answer = 1209 tickets

Example 2

A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate = 50 x 40 = 2000g

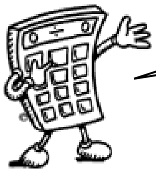
Calculate:

```
  42
 x48
 336   ← 8 x 42
1680   ← 40 X 42
2016
```

Answer = 2016g

Time 1

Time may be expressed in 12 or 24 hour notation.



12-hour clock

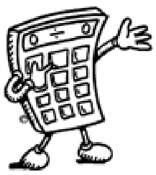
Time can be displayed on a clock face, or digital clock.

05: 15

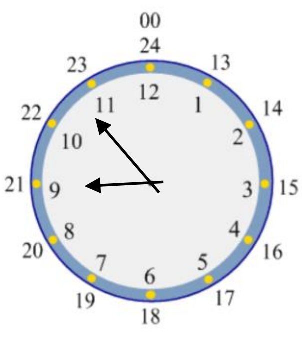
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



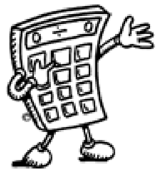
In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 … etc.



Examples

9.55 am ⟶ 0955 hours
3.35 pm ⟶ 1535 hours
12.20 am ⟶ 0020 hours
0216 hours ⟶ 2.16 am
2045 hours ⟶ 8.45 pm

Time 2



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
52 weeks
12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year.

Time Intervals

Example A train departs at 1020 and arrives at it's destination at 1305. How long did the journey take?

1020 → 1220 → 1300 → 1305

2 hours, 40 minutes, 5 minutes

= 2 hours 45 minutes

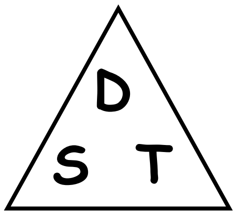
Time 3 – Speed, Distance and Time

For any given journey, the distance travelled depends on the speed and the time taken. If speed is constant, then the following formulae apply:

Distance = Speed x Time or $D = S \times T$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$



Example 1 Calculate the speed of a train which travelled 450 km in 5 hours

$$S = \frac{D}{T}$$

$$S = \frac{450}{5}$$

$S = 90$ km/h

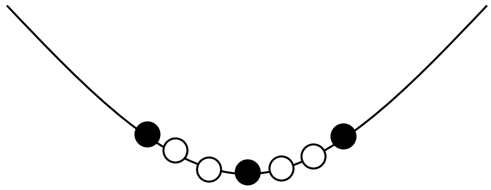
Example 2 A man runs at a speed of 10km/h for 3 hours. What distance does he cover?

$D = S \times T$
$D = 10 \times 3$
$D = 30$ km

Fractions 1

Understanding Fractions

Example
A necklace is made from black and white beads.

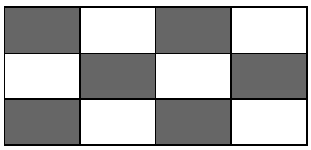


What fraction of the beads is black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ (three sevenths) of the beads are black.

Equivalent Fractions

Example
What fraction of the flag is shaded?



6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Examples of equivalent fractions

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{10}{20} \qquad \frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{10}{30}$$

$$\frac{3}{8}=\frac{6}{16}=\frac{9}{24}=\frac{15}{40}=\frac{30}{80} \qquad \frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{10}{25}=\frac{20}{50}$$

Fractions 2

Improper Fractions and Mixed Numbers

Example 1 Change the following to improper fractions

a $1\frac{2}{3}$

1 x 3 + 2

$= \frac{5}{3}$

b $5\frac{3}{6}$

5 x 6 + 3

$= \frac{33}{6}$

$= \frac{11}{2}$

*Remember to always simplify fractions when you can.

Example 2 Change the following to mixed numbers

a $\frac{7}{5}$

7 ÷ 5 = 1 r 2

$= 1\frac{2}{3}$

b $\frac{24}{9}$

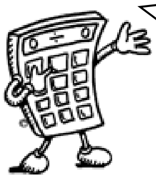
24 ÷ 9 = 2 r 6

$= 2\frac{6}{9}$

$= 2\frac{2}{3}$

Fractions 3

Simplifying Fractions



The top of a fraction is called the **numerator**; the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the **same number**.

Example 1

(a) $\frac{20}{25} = \frac{4}{5}$ (÷5 top and bottom)

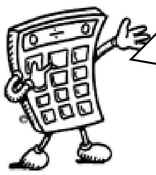
(b) $\frac{16}{24} = \frac{2}{3}$ (÷8 top and bottom)

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$

$\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator, then multiply by the numerator.

To find $\frac{3}{5}$ divide by 5 then multiply the answer by 3

<u>**OR**</u> multiply by 3 first then divide the answer by 5

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150
= £150 ÷ 5
<u>= £30</u>

Example 2 Find $\frac{3}{4}$ of 48

$\frac{1}{4}$ of 48
= 48 ÷ 4
= 12

so $\frac{3}{4}$ of 48
= 48 ÷ 4 x 3
= 12 x 3
= 36

"Divide by the bottom, multiply by the top"

Fractions 4

Adding and Subtracting Fractions

This just means the same number on the bottom of each fraction.

To add and subtract fractions they need to have a common denominator.

Example 1

a. $\frac{1}{5} + \frac{2}{5}$

$= \frac{1+2}{5}$

$= \frac{3}{5}$

b. $\frac{3}{8} - \frac{1}{8}$

$= \frac{3-1}{8}$

$= \frac{2}{8}$

$= \frac{1}{4}$

Always give your answer in the simplest form.

Example 2

$\frac{1}{2} + \frac{1}{3}$

$= \frac{1^{x3}}{2^{x3}} + \frac{1^{x2}}{3^{x2}}$

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{3+2}{6}$

$= \frac{5}{6}$

For different denominators you need to find the lowest common multiple.

Continued over the page...

When adding and subtracting mixed numbers deal with the whole numbers first.

Example 3

a.

$1\frac{1}{12} + 2\frac{3}{12}$

= $3\frac{1}{12} + \frac{3}{12}$

= $3\frac{1+3}{12}$

= $3\frac{4}{12}$

= $3\frac{1}{3}$

b.

$4\frac{1}{5} - 1\frac{1}{10}$

= $3\frac{1^{x2}}{5^{x2}} - \frac{1}{10}$

= $3\frac{2}{10} - \frac{1}{10}$

= $3\frac{2-1}{10}$

= $3\frac{1}{10}$

Lowest common multiple of 5 and 10 is 10 so we only need to change the first fraction.

Harder Examples!

a.

$2\frac{3}{4} + 3\frac{2}{5}$

= $5\frac{3^{x5}}{4^{x5}} + \frac{2^{x4}}{5^{x4}}$

= $5\frac{15}{20} + \frac{8}{20}$

= ★ $5\frac{23}{20}$

= $6\frac{3}{20}$

★ Change the improper fraction to a mixed number and add to the other whole numbers.

b.

$6\frac{1}{7} - 2\frac{2}{3}$

= $4\frac{1^{x3}}{7^{x3}} - \frac{2^{x7}}{3^{x7}}$

= $4\frac{3}{21} - \frac{14}{21}$

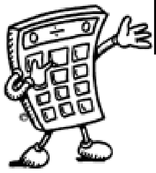
= ★ $3\frac{21}{21} + \frac{3}{21} - \frac{14}{21}$

= $3\frac{21+3-14}{21}$

= $3\frac{10}{21}$

★ If the second fraction is too big to subtract borrow from the whole numbers.

Percentages 1



Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal by dividing by 100.

36% means $\frac{36}{100}$

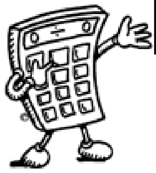
$$36\% = \frac{36}{100} = \frac{9}{25} = 0.36$$

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0·01
10%	$\frac{1}{10}$	0·1
20%	$\frac{1}{5}$	0·2
25%	$\frac{1}{4}$	0·25
$33^1/_3$%	$\frac{1}{3}$	0·333…
50%	$\frac{1}{2}$	0·5
$66^2/_3$%	$\frac{2}{3}$	0·666…
75%	$\frac{3}{4}$	0·75

Percentages 2



There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non- Calculator Methods

Method 1   Using Equivalent Fractions

Example   Find 25% of £640

25% of £640 $= \frac{1}{4}$ of £640 = £640 ÷ 4 = £160

Method 2   Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example   Find 9% of 200g

1% of 200g = $\frac{1}{100}$ of 200g = 200g ÷ 100 = 2g

so 9% of 200g = 9 x 2g = 18g

Method 3   Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example   Find 70% of £35

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3·50

so 70% of £35 = 7 x £3·50 = £24·50

Percentages 3

Non- Calculator Methods (continued)

The previous 2 methods can be combined so as to calculate any percentage.

Example Find 23% of £15000

10% of £15000 = £1500 **so** 20% = £1500 x 2 = £3000
1% of £15000 = £150 **so** 3% = £150 x 3 = £450

23% of £15000 = £3000 + £450 = <u>£3450</u>

Finding VAT (without a calculator)

Value Added Tax (VAT) = 20%
To find VAT, firstly find 10% and then multiply but 2.

Example Calculate the total price of a computer which costs £650 excluding VAT

10% of £650 = £65 (divide by 10)
20% of £650 = £130 (multiply previous answer by 2)

<u>**so**</u> 20% of £650 = £130

Percentages 4

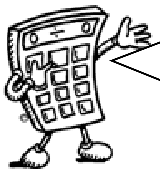
Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

23% = 0·23

so 23% of £15000 = $\frac{23}{100}$ x £15000 = 0·23 x £15000 = £3450



We do not use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals.

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

19% = $\frac{19}{100}$ = 0·19 so Increase = 0·19 x £236000

= £44840

Value at end of year = original value + increase

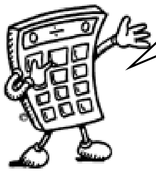
= £236000 + £44840

= £280840

The new value of the house is £280840

Percentages 5

Making a percentage



To make a percentage of a total, first make a fraction, and then multiply by 100.

Example 1 There are 30 pupils in Class 3A3. 18 are girls.
What percentage of Class 3A3 are girls?

$\frac{18}{30} \times 100 = 18 \div 30 \times 100 = 60\%$

60% of 3A3 are girls

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

Score = $\frac{36}{44} \times 100 = 36 \div 44 \times 100 = 81{\cdot}818..\%$

$= 82\%$ (rounded)

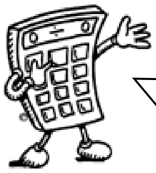
Example 3 In class 1X1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

Total number of pupils = 14 + 6 + 3 + 2 = 25
6 out of 25 were blonde, so,

$\frac{6}{25} \times 100 = 6 \div 25 \times 100 = 24\%$

24% were blonde.

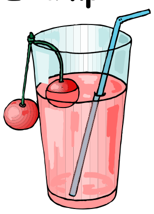
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is R : B : G
5 : 7 : 8

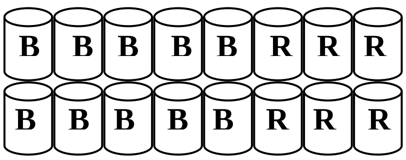
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



B : R
= 10 : 6
= 5 : 3

To simplify a ratio, divide each figure in the ratio by a common factor.

Ratio 1 (cont)

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6
= 2:3
Divide each figure by 2

(b) 24:36
= 2:3
Divide each figure by 12

(c) 6:3:12
= 2:1:4
Divide each figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
×5 ↓	↓ × 5
15	**10**

So the chocolate bar will contain 10g of nuts.

Ratio 2

Sharing in a given ratio

Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$3 + 2 = 5$

Step 2 Divide the total by this number to find the value of each part

$90 \div 5 =$ £18

Step 3 Multiply each figure by the value of each part

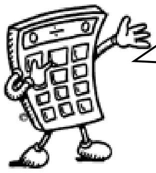
Lauren 3 x £18 = £54
Sean 2 x £18 = £36

Step 4 Check that the total is correct

£54 + £36 = £90

So Lauren received £54 and Sean received £36.

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1
A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
90	**4500**

(×3 on both sides)

The factory would produce 4500 cars in 90 days.

Example 2
5 adult tickets for the cinema cost £27·50. How much would 8 tickets cost?

Find the cost of 1 ticket →

Tickets	Cost
5	£27·50
1	£5·50
8	£44·00

Working:

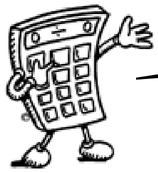
£27·50 ÷ 5 = £5·50

£5·50 × 8 = £44·00

The cost of 8 tickets is £44

Information Handling : Tables

It is sometimes useful to display information in graphs, charts or tables.



Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

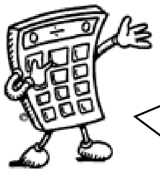
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20	𝍷𝍷	2
21 - 25	𝍸 𝍷𝍷	7
26 - 30	𝍸 𝍷𝍷𝍷𝍷	9
31 - 35	𝍸	5
36 - 40	𝍷𝍷𝍷	3
41 - 45	𝍷𝍷	2
46 - 50	𝍷𝍷	2

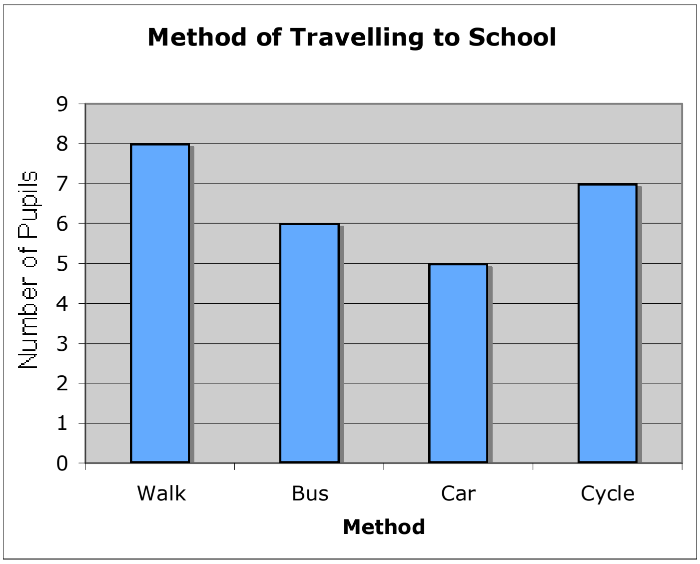
Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs and Histograms



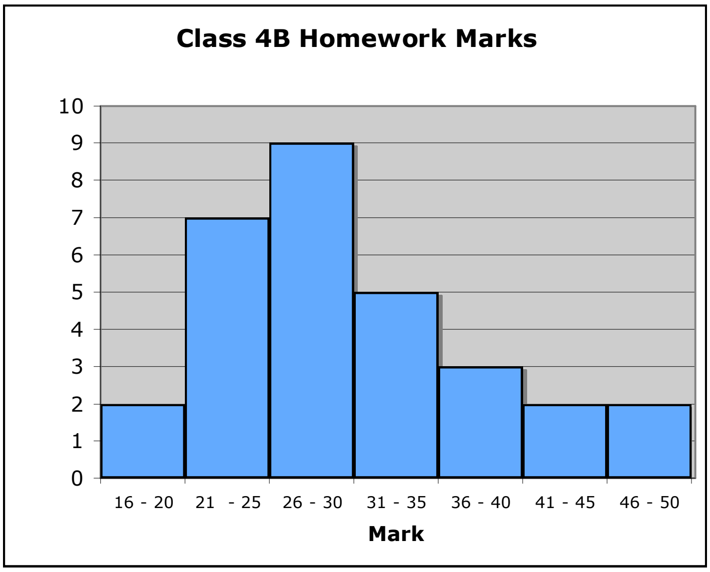
Bar graphs and histograms are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 This **bar graph** shows how a group of pupils travelled to school.



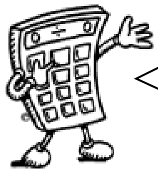
Notice that the bar graph has gaps between the information and is used for categories (meaning that the order can be changed).

Example 1 This **histogram** shows the homework marks for Class 4B.



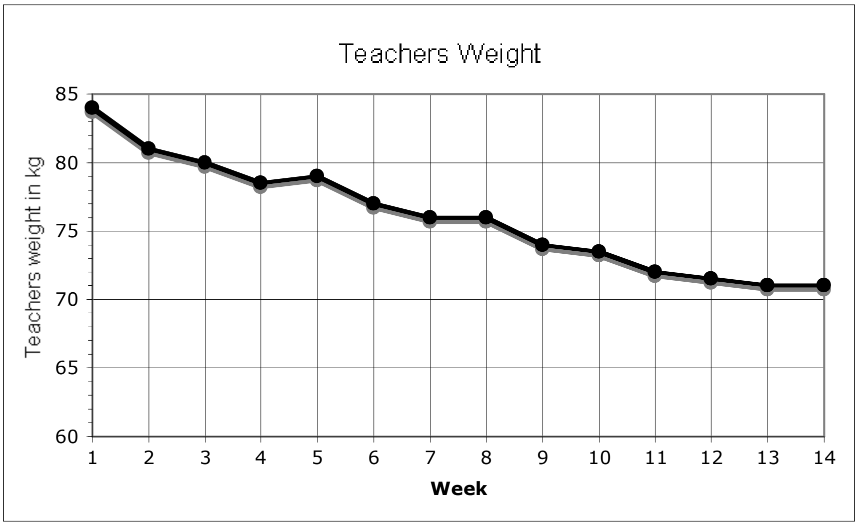
Notice that the histogram is used for class intervals (it must remain in this order) and has no gaps.

Information Handling : Line Graphs



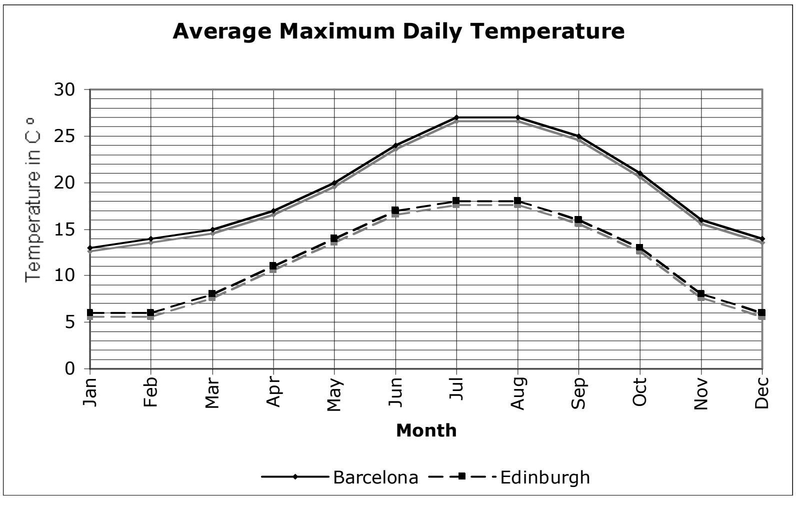
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows a teachers weight over 14 weeks as he follows an exercise programme.

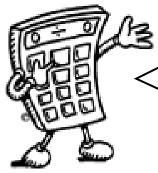


The trend of the graph is that his weight is decreased over the 14 weeks he trained.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



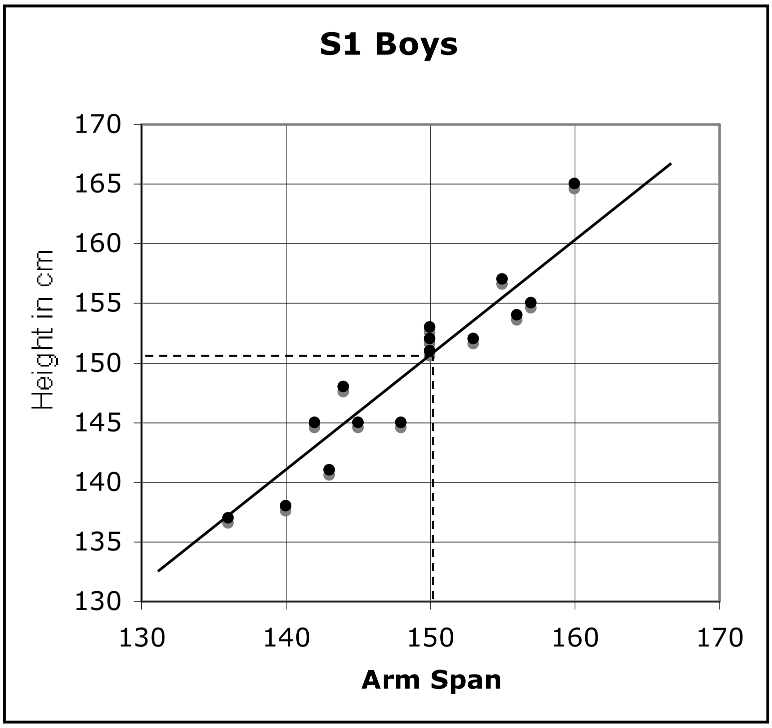
Information Handling : Scatter Graphs



A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

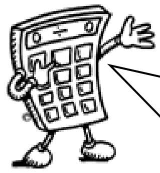


The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

The line drawn is called the line of best fit, it has the same amount of points above as below. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

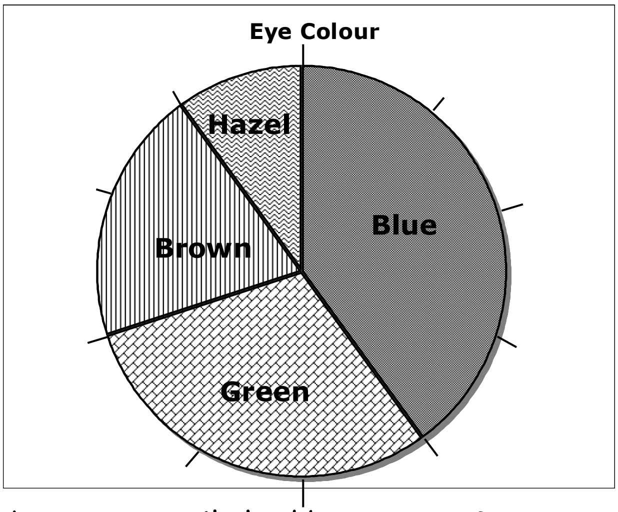
Note that in some subjects, it is a requirement that the axes start from zero.

Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

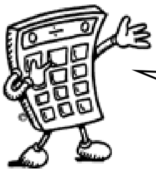
If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72°.
so the number of pupils with brown eyes
$= \frac{72}{360} \times 30 = 6$ pupils.
If finding all of the values, you can check your answers - the total should be 30 pupils.

Each sector should be clearly labelled.

Information Handling : Pie Charts 2

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

Example: In a survey about television programmes, a group of people were asked what their favourite soap was. Their answers are given in the table below. Draw a pie chart to illustrate the information.

Soap	Number of people
Eastenders	28
Coronation Street	24
Emmerdale	10
Hollyoaks	12
None	6

Total number of people = 80

Eastenders $= \frac{28}{80} \rightarrow \frac{28}{80} \times 360^\circ = 126^\circ$

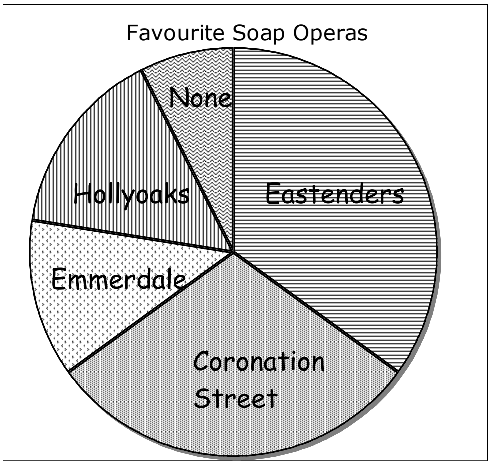
Coronation Street $= \frac{24}{80} \rightarrow \frac{24}{80} \times 360^\circ = 108^\circ$

Emmerdale $= \frac{10}{80} \rightarrow \frac{10}{80} \times 360^\circ = 45^\circ$

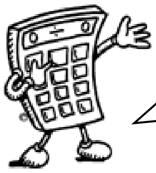
Hollyoaks $= \frac{12}{80} \rightarrow \frac{12}{80} \times 360^\circ = 54^\circ$

None $= \frac{6}{80} \rightarrow \frac{6}{80} \times 360^\circ = 27^\circ$

Check that the total = 360°



Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value – the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value – Lowest value

Example Class 1A4 scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

Mean = $\frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14}$

= $\frac{102}{14} = 7.285...$ = 7·3 to 1 decimal place

Ordered values: 4, 5, 5, 6, 7, 7, ↓ 7, 7, 8, 8, 9, 9, 10, 10

Median = 7

Mode = 7

Range = 10 – 4 = 6

More Examples on how to calculate the Median

2, 2, 3, ↓ 5, 6, 7

Median = 4

6, 7, 7, ↓ 8, 10, 11

Median = 7·5

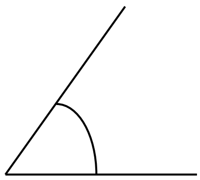
1, 2, 5, ↓ 11, 12, 15

Median = $\frac{5+11}{2}$

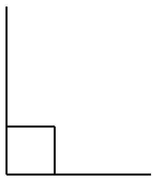
= 8

Angles

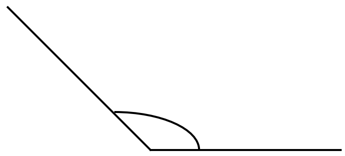
Types of angles



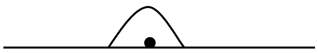
Acute
1° to 89°



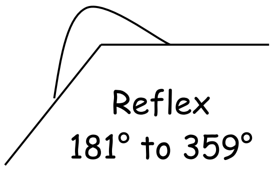
Right Angle
90°



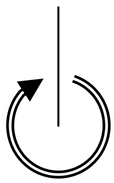
Obtuse
90° to 179°



Straight Line
180°

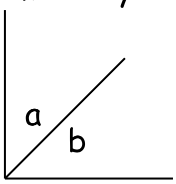


Reflex
181° to 359°



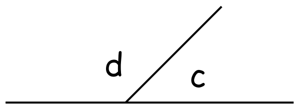
1 full turn or
revolution
360°

Complementary Angles
When two angles can fit together to make a right angle we say they are complementary



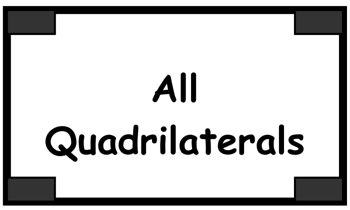
a + b = 90°

Supplementary Angles
When two angles fit together to make a straight angle we say they are supplementary

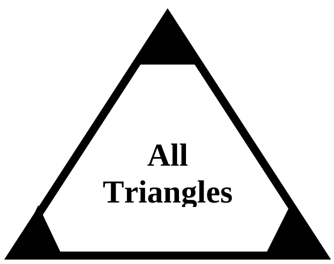


c + d = 180°

Angles inside shapes



All internal angles in a quadrilateral add up to 360°



All internal angles add up to 180°

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Chunking	Splitting a number into chunks to work with
Compensating	Adding on to make a number more manageable then taking away at the end.
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (÷)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$

Greater than or equal to (≥)	Is bigger than **<u>OR</u>** equal to.
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.
Less than or equal to (≤)	Is smaller than **<u>OR</u>** equal to.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p32)
Median	Another type of average - the middle number of an ordered set of data (see p32)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract. (sometimes referred to as take away)
Mode	Another type of average – the most frequent number or category (see p32)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS (see p9)
Partitioning	Splitting a number into it's place value parts.
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Polygon	A plane shape (2-D Shape) which has three or more straight sides.

Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Quadrilateral	A polygon with four sides
Quotient	The number resulting by dividing one number by another. Example: 20 ÷ 2 = 10, the quotient is 10
Range	Subtract the lowest number from the highest number to find the Range. This indicates how variable the data is.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Square Numbers	A number that results from multiplying another number by itself.
Total	The sum of a group of numbers (found by adding).

Useful Websites

There are many valuable online sites that can offer help and more practice. Many are presented in a games format to make it more enjoyable for your child.

The following sites have been found to be useful.

https://www.sumdog.com/

http://mathsrevision.com/

https://www.mymaths.co.uk/

https://www.coolmath4kids.com/

http://www.mathsisfun.com/

www.amathsdictionaryforkids.com

http://resources.woodlands.kent.sch.uk/

http://www.bbc.co.uk/education

www.topmarks.co.uk

www.primaryresources.co.uk/maths

There are also some great apps out there that can be used to develop Maths skills. A good selection can be found at

http://www.parents.com/kids/education/math-and-science/best-math-apps-for-kids/